Surface normal estimation of thin transparent objects from polarization of transmitted light

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Abstract

Shape estimation of transparent objects is usually difficult since the light transmits. The proposed method estimates the surface normal of objects through polarization analysis of light. Most technique that use polarization for shape estimation analyzes the reflected light. However, in order to observe the reflection over the whole surface, we have to illuminate the object from every direction. In this paper, we propose a novel method for estimating the surface shapes of transparent objects by analyzing the polarization state of the transmitted light. The target objects are thin transparent objects, such as bottles or glasses. Thin transparent objects less refract, thus, existing methods that analyze the light transport for shape estimation fail. On the other hand, our polarization-based method can estimate the unique surface normal of thin transparent object.

Figure 1. Light path of refraction: (a) Solid transparent object and (b) thin transparent object.

1. Introduction

This paper presents a novel method for estimating the surface shape of transparent objects by analyzing the polarization of transparent objects. Many methods [1] compute the transparent shape by analyzing the refraction of the light ray; thus, they cannot measure a thin transparent object, which causes less refraction (Fig. 1).

Polarization is one of the characteristics that can be used to obtain a smooth surface normal [2–11]. Existing methods [8, 9] analyzed the polarization state of the reflected light, and estimated the surface of a transparent object.

We propose a method for estimating the surface normal using polarization analysis. The polarization information of the object is obtained from the transmitted light using a polarization imaging camera. Previous work [8, 9] estimated the shape of transparent object from the polarization of reflected light. However, the zenith angle of the surface normal and the degree of polarization of the reflected light has 1-to-2 correspondence. Also, the light should be illuminated f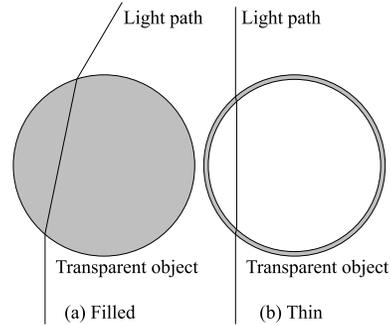rom every direction. On the other hand, our method analyzes the transmitted light. In this case, the zenith angle of the surface normal and the degree of polarization of the transmitted light has 1-to-1 correspondence. Also, a single light source is enough. Especially, our work measures the light which transmits the object 4 times. The degree of polarization of transmission 4 times is higher than that of transmission once, thus, our method is more robust than other methods.

2. Shape from polarization of transmitted light

We explain only linear polarization since circular polarization is not related to our method. DOP (degree of polarization) is one of the metrics used to represent the polarization state of light. Its value varies from 0 to 1, with 1 representing perfectly polarized light and 0 representing unpolarized light. The maximum light observed while rotating the polarizer is denoted as $I_{\max}$, and the minimum light is denoted as $I_{\min}$. The polarizer angle at which $I_{\max}$ is observed is called the phase angle ψ. The DOP is defined

as follows.

$$\rho = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} . \tag{1}$$

We set the camera at the end of the z-axis. The surface normal $\mathbf{n} = (n_x, n_y, n_z)$ can be represented in polar coordinate system. The zenith angle θ of the surface normal is the angle between the surface normal and the z-axis. The azimuth angle ϕ is defined in xy-plane, and it is an angle between the x-axis and the surface normal projected to the xy-plane.

Suppose that the surface of the target dielectric object is optically smooth. The angle between the surface normal and the transmitted light path is denoted as θ.

The plane consisting of the transmitted light and surface normal vectors is called the POT (plane of transmission). The incident vector is also coplanar with the POT. The orientation of the POT is denoted as ϕ, which is defined on a certain xy-plane and is defined as an angle between x-axis and the POT projected on xy-plane. The POT angle ϕ coincides with the azimuth angle ϕ of the surface normal. Also, the phase angle ψ or its opposite $\psi + 180°$ coincides with the azimuth angle ϕ. Namely, $180°$ ambiguity exists for determining the unique azimuth angle.

The intensity ratio of transmitted light to incident light is called intensity transmissivity T. Subscripts p and s represent the components parallel and perpendicular to POT, respectively. Intensity transmissivity for dielectric transparent material is given as follows:

$$T_p = \frac{\sin 2\theta \sin 2\theta'}{\sin^2(\theta + \theta') \cos^2(\theta - \theta')} , \tag{2}$$

$$T_s = \frac{\sin 2\theta \sin 2\theta'}{\sin^2(\theta + \theta')} . \tag{3}$$

Here, θ and θ' are the transmission angle and the incidence angle, respectively, or vice versa.

As shown in Eq. (2)–(3), $T_p \geq T_s$ holds, thus, the DOP (Eq. (1)) of the transmitted light is represented as follows.

$$\rho = \frac{T_p - T_s}{T_p + T_s} . \tag{4}$$

We set the target object on the light box, and observe the transmitted light by polarization camera. We calculate the azimuth angle from the phase angle and the zenith angle from the DOP. Our algorithm is pixel-based, and thus, the result for each pixel is not affected by neighboring pixels. We assume that the index of refraction is known.

The azimuth angle ϕ is ambiguous, where it might be ψ or $\psi + 180°$. We assume that the object is convex in z-axis, where the peak is the center pixel position $(\bar{x}, \bar{y})$.

Vector $(p_x, p_y) = (\cos\psi, \sin\psi)$ represents the orientation of the phase angle ψ. Vector $(q_x, q_y) = (\cos(\psi + \pi), \sin(\psi + \pi))$ represents the orientation of the phase angle $\psi + 180°$. Vector $(v_x, v_y) = (x - \bar{x}, y - \bar{y})$ is the vector
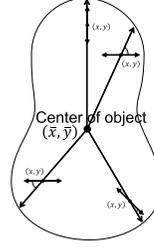
Figure 2. Center pixel position of the target object.
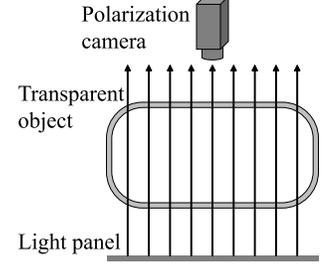
Figure 3. The geometry of our measurement.

from the image center (object center) $(\bar{x}, \bar{y})$ to the pixel of interest (x, y) (Fig. 2).

As a result, the $180°$ ambiguity of the azimuth angle can be solved by calculating the dot product between the (p_x, p_y) and (v_x, v_y).

$$\phi = \begin{cases} \psi & \text{if } (p_x, p_y) \cdot (v_x, v_y) > 0 \\ \psi + \pi & \text{otherwise} \end{cases} . \tag{5}$$

This work measures the bottles and pots of glasses or plastics. As is shown in Fig. 3, the light transmits 4 times. The light transmits twice at the front surface, and also it transmits twice at the rear surface.

Our algorithm does not consider refraction. The infinitely thin object does not cause refraction, thus, the target objects of our method are thin objects.

Also, we assume that the rear surface and the front surface are in mirror reflection along z-axis (Fig. 4). For this case, the angle between each surface normal and each light path is θ_1. As a result, the transmissivity for 4 surfaces become the same. Our method estimates the surface normal of the closest surface. However, due to the geometrical relation, all 4 normals of all surfaces are obtained.

$$\begin{aligned} \mathbf{n} &= (n_x, n_y, n_z) , \\ -\mathbf{n} &= (-n_x, -n_y, -n_z) , \\ \bar{\mathbf{n}} &= (n_x, n_y, -n_z) , \\ -\bar{\mathbf{n}} &= (-n_x, -n_y, n_z) . \end{aligned} \tag{6}$$

Three lines of Fig.5 are the DOP of reflected light, once transmitted light, and four-times transmitted light. This figure shows that DOP of reflected light has 1-to-2 correspondence, but DOP of transmitted light has 1-to-1 correspondence. Also, the 1-to-1 correspondence of 4 times transmission is wider than that of once transmission. This proves the robustness of our method.
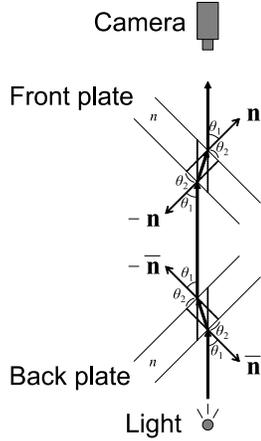
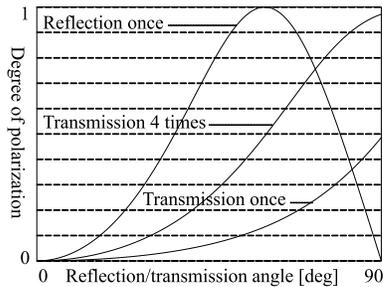**Figure 4. Surface normal and transmitted angle of a thin object.**

Figure 5. Degree of polarization of 1 reflection, 1 transmission, and 4 transmissions.

3. Experimental result

The experiment is performed in the dark room (Fig. 6). The target object is set on the light box. We fix the light, the target object, and the camera. We take one image using the polarization camera.

Since the ground truth of a sphere is known, we used it for evaluation. Input data are shown in Fig. 7, and the output data are shown in Fig. 8. The error is shown in Fig. 9. The average angular error between the true surface normal and the estimated surface normal was 0.161 [rad]. Our method successfully estimated the spherical shape of a transparent object. On the other hand, the boundary of the sphere has high error. We assume that the surfaces are parallel, thus, the area where the curvature is high fails.

The results of real transparent objects are shown in Fig. 10–11. The photo of the objects are shown in Fig. 10–11 (a). The input data are shown in Fig. 10–11 (b)(c). The estimated surface normals are shown in Fig. 10–11 (d).
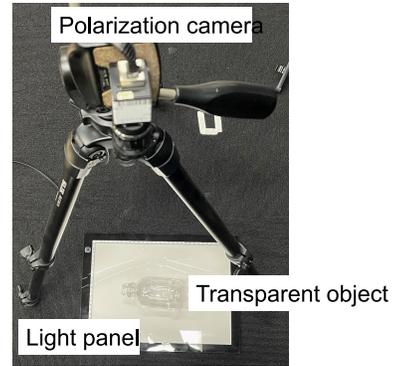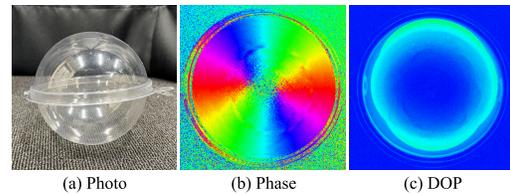

Figure 6. Experimental setup.

(a) Photo   (b) Phase   (c) DOP

Figure 7. Thin sphere made of transparent plastics: (a) Target object, (b) phase angle, and (c) degree of polarization.

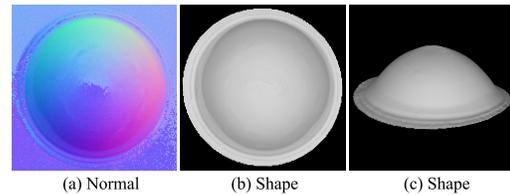(a) Normal   (b) Shape   (c) Shape

Figure 8. Thin sphere made of transparent plastics: (a) Estimated surface normal, and (b)(c) rendered 3D shape.
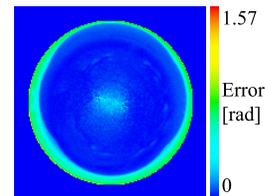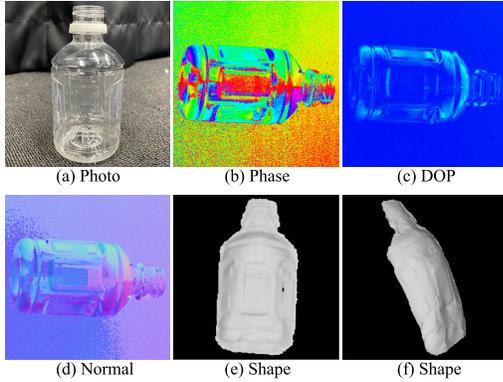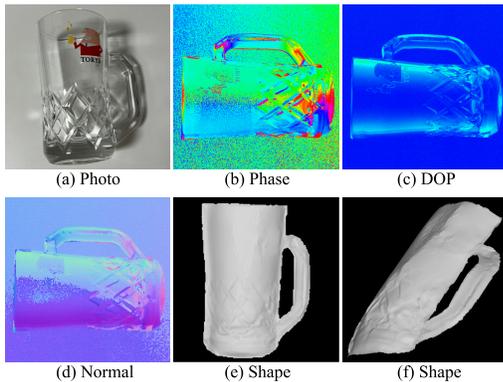

Figure 9. Error map of the surface normal for the transparent sphere.

Figure 10. Thin bottle made of transparent plastics: (a) Target object, (b) phase angle, (c) degree of polarization, (d) estimated surface normal, and (e)(f) rendered 3D shape.

Figure 11. Thin bottle made of transparent glass: (a) Target object, (b) phase angle, (c) degree of polarization, (d) estimated surface normal, and (e)(f) rendered 3D shape.

The integrated shapes are shown in Fig. 10–11 (e)(f). Our method successfully obtained the detailed structure of the transparent objects. However, we assume that the object is convex along the z-axis where the peak is the center of the image. This assumption is not always satisfied, and the concave parts are less precise.

4. Conclusion

In this paper, we have proposed a novel method for estimating the surface shape of transparent objects. We estimated the surface normal of the object by observing the polarization information from a single viewpoint under a sin-

gle light source. The ambiguity problem of the azimuth angle of surface normal is solved by assuming that the object is convex. The zenith angle of surface normal is uniquely determined because the degree of polarization of the transmission has 1-to-1 correspondence to the zenith angle. Our method can successfully determine the surface normal even though we did not consider the refraction.

References

[1] I. Ihrke, K. N. Kutulakos, H. P. A. Lensch, M. Magnor, and W. Heidrich, "Transparent and specular object reconstruction," *Computer Graphics Forum*, **29**(8), 2400–2426 (2010).

[2] G. A. Atkinson and E. R. Hancock, "Recovery of surface orientation from diffuse polarization," *IEEE Transactions on Image Processing*, **15**(6), 1653–1664 (2006).

[3] G. A. Atkinson and E. R. Hancock, "Shape estimation using polarization and shading from two views," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **29**(11), 2001–2017 (2007).

[4] G. C. Guarnera, P. Peers, P. Debevec, and A. Ghosh, "Estimating surface normals from spherical Stokes reflectance fields," in *Computer Vision - ECCV2012. Workshops and Demonstrations*, 340–349 (2012).

[5] C. P. Huynh, A. Robles-Kelly, and E. R. Hancock, "Shape and refractive index from single-view spectro-polarimetric images," *International Journal of Computer Vision*, **101**, 64–94 (2013).

[6] A. Kadambi, V. Taamazyan, B. Shi, and R. Raskar, "Polarized 3D: High-quality depth sensing with polarization cues," in *Proceedings of IEEE International Conference on Computer Vision*, 3370–3378 (2015).

[7] D. Miyazaki, R. T. Tan, K. Hara, and K. Ikeuchi, "Polarization-based inverse rendering from a single view," in *Proceedings of IEEE International Conference on Computer Vision*, 982–987 (2003).

[8] D. Miyazaki, M. Kagesawa, and K. Ikeuchi, "Transparent surface modeling from a pair of polarization images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **26**(1), 73–82 (2004).

[9] D. Miyazaki and K. Ikeuchi, "Shape estimation of transparent objects by using inverse polarization raytracing," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **29**(11), 2018–2030 (2007).

[10] S. Rahmann and N. Canterakis, "Reconstruction of specular surfaces using polarization imaging," in *Proceedings of IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 149–155 (2001).

[11] L. B. Wolff and T. E. Boult, "Constraining object features using a polarization reflectance model," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **13**(7), 635–657 (1991).