Hue enhancement for dichromats using Poisson equation

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Dichromats recognize colors using two out of three cone cells; L, M, and S. In order to extend the ability of dichromats to recognize the color difference, the authors propose a method to expand the color difference when observed by dichromats. Most methods analyze the color in color space, while their method analyze the color in image space. Namely, they analyze the color between the neighboring pixels not in intensity space but in chromaticity space, and form a Poisson equation. Solving the Poission equation results in an image for dichromats whose relative color difference between neighboring pixels is as same as the image observed by trichromats.

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I. INTRODUCTION

Red, green, and blue colors are detected by three kinds of cone cells embedded in the retina. Dichromats use two of them to recognize colors. In this article, we propose a method to enhance the visibility of dichromats.

Enhancing the visibility of color image for dichromats is an important research field[1–21]. For example, Rasche et al.[22] transformed the color space with homographic projection so that the color difference of dichromats becomes similar to that of trichromats. Nakauchi and Onouchi[23] applied a clustering algorithm in color space and they stretched the color difference so that each cluster becomes far apart. Kuhn et al.[24] employed spring-mass model in order to make the color difference of dichromats to be as same as that of trichromats. Tanaka et al.[25] gave a closed-form solution to the cost function, where the color difference between neighboring pixels becomes similar to a certain value, which is calculated from the color difference of neighboring pixels under trichromatic view. Miyazaki et al.[26] defined a hue for dichromats and converted the trichromat's hue to dichromat's hue so that the hue difference will be the same among these people.

Our approach is quite different from existing methods. Most methods first map all pixel colors in color space such as RGB, HSV, XYZ, LMS, L*a*b*, etc., and next, they deform the color space or deform the clusters of mapped points so that it satisfies the required condition. On the other hand, our method analyzes the color difference between neighboring pixels. Namely, our method analyzes not in color space but in image space (*i.e.*, pixel coordinates). Unlike Tanaka et al.[25], which analyzes intensity instead of hue, we analyze hue instead of intensity. We preserve the color difference recognized by trichromats, and provide the same color difference to dichromats as that of trichromats. We formulate the Poisson equation so that the relative color difference between neighboring pixels will be preserved. Some methods[27,28] also solve the Poisson equation in order to enhance the visibility of dichromats. These methods form the Poisson equation in RGB intensity space, while our method forms in xy-chromaticity space. As a result, our method exaggerates the color difference between neighboring pixels. Also, dichromats perceive the color difference of our output image as the same as trichromats do.

Section II explaines the basic theory, and Section III shows our method. In Section IV, we show some results, and we also discuss the disadvantage of our method. Section VI concludes our article.

II. HUE FOR DICHROMATS

The color value which dichromats perceive can be calculated as follows. RGB value is first converted to CIE-XYZ value, and after that, it is converted to LMS value. The LMS represents the sensitivity of cone cells. The procedures to calculate the LMS values of dichromats are shown in some literature, such as Judd[29] and Brettel et al.[30]. In this article, we follow Judd[29]. The conversion formula for protanopia is shown below.

$$\begin{pmatrix} L_p \\ M_p \\ S_p \end{pmatrix} = \begin{pmatrix} 0.0 & 2.02 & -2.52 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \begin{pmatrix} L \\ M \\ S \end{pmatrix} . \tag{1}$$

And, the conversion formula for deuteranopia is shown below.

$$\begin{pmatrix} L_d \\ M_d \\ S_d \end{pmatrix} = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.49 & 0.0 & 1.25 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \begin{pmatrix} L \\ M \\ S \end{pmatrix} . \tag{2}$$

In xy-diagram calcualted from CIE-XYZ value, the white color is placed in $(x, y) = (0.33, 0.33)$ for trichromats. First, we define the hue α of trichromats as an angle defined in xy-plane (Figure 1 (a)). The trichromatic hue α is defined as an angle around the white point $(x, y) = (0.33, 0.33)$. We define the 0° of α to be the direction of -45°. The hue angle α of a certain color (x, y) is calculated as follows.

$$\alpha = \frac{\pi}{4} + \tan^{-1} \frac{y - 0.33}{x - 0.33} . \tag{3}$$

We also define the hue β of dichromats (Fig. 1(b, c)), which has strong relation with the ***L*a*b**** hue of trichromats[26]. Our definition of hue is mathematically convincing[26]. Following Judd[29], the white point of protanopia is $(x, y) = (0.747, 0.253)$ and that of deuteranopia is $(x, y) = (1.000, 0.000)$. We define the hue β rotating around these white points, where it ranges from 140° direction to 200° direction for protanopia and ranges from 140° direction to 170° direction for deuteranopia. The angles of these ranges are carefully chosen so that the defined hue distributes inside the color gamut. The hue angle β of protanopia is calculated as follows:

$$\beta = \frac{\pi}{180} \left(140 + \frac{\alpha}{2\pi} (200 - 140) \right) , \tag{4}$$

and the hue angle β of deuteranopia is calcuted as follows:

$$\beta = \frac{\pi}{180} \left(140 + \frac{\alpha}{2\pi} (170 - 140) \right) . \tag{5}$$

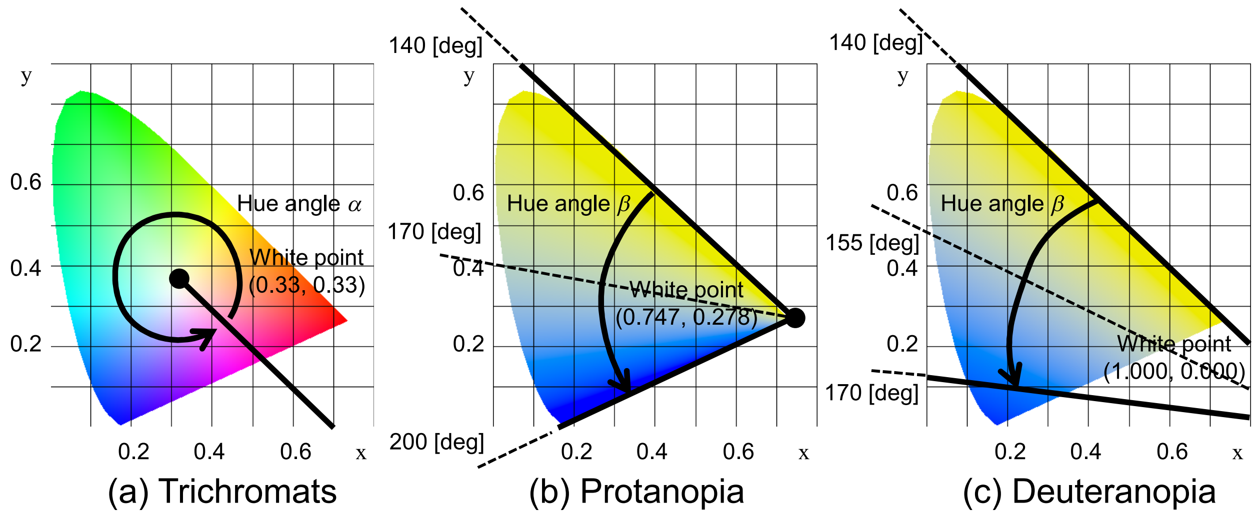


FIG. 1. Definition of hue of (a) trichromats, (b) protanopia, and (c) deuteranopia.

Here, the hue angle α ranges from 0 to 2π. The direction of $0°$ in hue angle α is casually defined (i.e., $45°$); however, it does not matter for our purpose as explained mathematically in Section III. Section III explains that our method use the relative value for representing the hue instead of the absolute value. This is because that the hue of trichromats ranges from 0 to 2π, but the 2π is cyclically connected to 0, because that the hue of dichromats ranges in a limited range.

Most of the color spaces are defined for trichromats. These color spaces such as *L*a*b** or YCbCr assumes that human eye is most sensitive to green channel compared to red and blue channels. However, deuteranopia lacks the sensitivity to green color and has only the sensitivity to red and blue color. Namely, the precondition of trichromats's color space is often invalid when considering dichromats. Therefore, this article uses the hue defined in this section[26].

III. COLOR ENHANCEMENT FOR DICHROMAT

The purpose of the method is to enhance the visibility of the image for dichromats. As shown in Sec. II, we represent the color as the hue angle shown in Fig. 1.

sRGB value of the input image is converted to CIE-XYZ. After that, CIE-XYZ value is converted to xy-chromaticity as follows:

$$\tilde{x} = \frac{\tilde{X}}{\tilde{X} + \tilde{Y} + \tilde{Z}}\,, \tag{6}$$

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{X} + \tilde{Y} + \tilde{Z}}\,, \tag{7}$$

$$\tilde{z} = \frac{\tilde{Z}}{\tilde{X} + \tilde{Y} + \tilde{Z}}\,. \tag{8}$$

The vector from the white point $(1/3, 1/3)$ of xy chromaticity to the chromaticity of image pixel is represented as Eq. (9).

$$\mathbf{a}(j,i) = \begin{pmatrix} \tilde{x}(j,i) - 0.33 \\ \tilde{y}(j,i) - 0.33 \\ 0 \end{pmatrix}\,. \tag{9}$$

Here, we use (j, i) for representing the x and y components of pixel position represented in Euclidean coordinates with x and y axes.

We denote the 4-neighbor pixel position as $(j + \Delta j, i + \Delta i)$, where the integer values Δj and Δi obey $|\Delta j| + |\Delta i| = 1$. The color vectors of neighboring pixels are also calculated as Eq. (10).

$$\tilde{\mathbf{a}}(j+\Delta j, i+\Delta i) = \begin{pmatrix} \tilde{x}(j+\Delta j, i+\Delta i) - 0.33 \\ \tilde{y}(j+\Delta j, i+\Delta i) - 0.33 \\ 0 \end{pmatrix}\,. \tag{10}$$

We normalize these vectors and denote them as $\hat{\mathbf{a}}(j, i)$ and $\hat{\mathbf{a}}(j + \Delta j, i + \Delta i)$ (Figure 2). We denote the cross product of these two vectors as $\mathbf{a}$.

$$\mathbf{a}(j+\Delta j, i+\Delta i) = \hat{\mathbf{a}}(j+\Delta j, i+\Delta i) \times \hat{\mathbf{a}}(j,i)\,. \tag{11}$$

Calculating the arcsine of $\mathbf{a}$ results in the signed angle between $\hat{\mathbf{a}}(j + \Delta j, i + \Delta i)$ and $\hat{\mathbf{a}}(j, i)$. We denote this angle as $\Delta\alpha(j + \Delta j, i + \Delta i)$ (Figure 3).

$$\Delta\alpha(j+\Delta j, i+\Delta i) = \sin^{-1}(\mathbf{a}(j+\Delta j, i+\Delta i))\,. \tag{12}$$

The discretized representation of the Laplacian of the hue angle $\tilde{\beta}$ for dichromats is as follows:

$$\begin{aligned} \triangle\tilde{\beta}(j,i) = -\Big\{&\tilde{\beta}(j,i) - \frac{1}{4}\tilde{\beta}(j-1,i) \\ &-\frac{1}{4}\tilde{\beta}(j+1,i) - \frac{1}{4}\tilde{\beta}(j,i-1) - \frac{1}{4}\tilde{\beta}(j,i+1)\Big\}\,. \end{aligned} \tag{13}$$

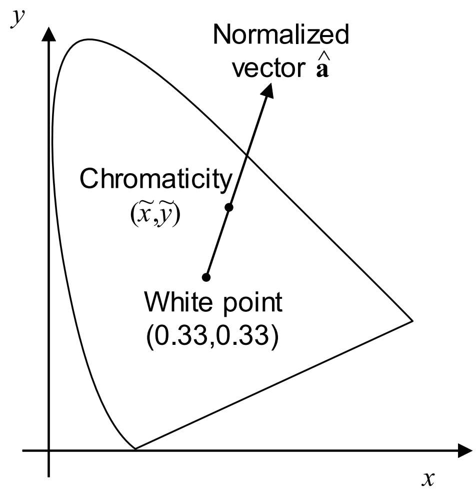


FIG. 2. Chromaticitiy vector.

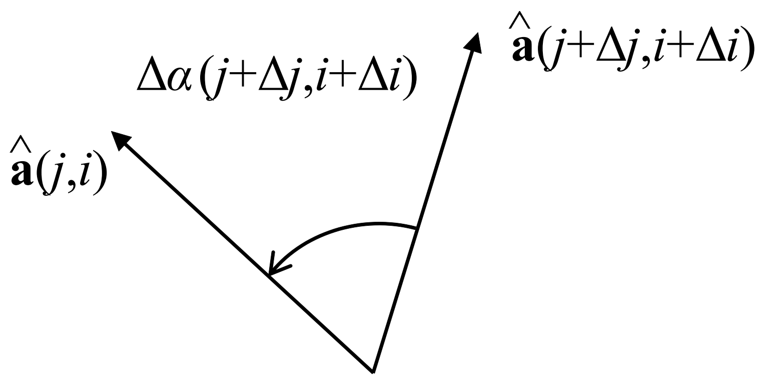


FIG. 3. Relative hue angle between two neighboring pixels.

Same goes to α.

$$\triangle\alpha(j,i) = -\Big\{\alpha(j,i) - \frac{1}{4}\alpha(j-1,i) \\ -\frac{1}{4}\alpha(j+1,i) - \frac{1}{4}\alpha(j,i-1) - \frac{1}{4}\alpha(j,i+1)\Big\}. \tag{14}$$

Eq. (14) is also represented as follows:

$$\begin{aligned}&\triangle\alpha(j,i) = -\Big\{\frac{1}{4}\left(\alpha(j,i)-\alpha(j-1,i)\right)\\&+\frac{1}{4}\left(\alpha(j,i)-\alpha(j+1,i)\right)\\&+\frac{1}{4}\left(\alpha(j,i)-\alpha(j,i-1)\right)\\&+\frac{1}{4}\left(\alpha(j,i)-\alpha(j,i+1)\right)\Big\}\,. \end{aligned} \tag{15}$$

If we simply subtract two angles, the calculation will fail since the angle has a cycle of $360°$. For example, $5°$ minus $355°$ should be $10°$, not $-350°$. If we convert an angle to a vector and calculate the angle between two vectors, this problem will not occur. Unlike dot product of two vectors, cross product of two vectors can calculate the angle with signed value. Using Eq. (11), Eq. (15) can be rewritten as follows:

$$\begin{aligned}&\triangle\alpha(j,i) = -\Big\{\frac{1}{4}\Delta\tilde{\alpha}(j-1,i)+\frac{1}{4}\Delta\tilde{\alpha}(j+1,i)\\&+\frac{1}{4}\Delta\tilde{\alpha}(j,i-1)+\frac{1}{4}\Delta\tilde{\alpha}(j,i+1)\Big\}\,. \end{aligned} \tag{16}$$

The difference of hue angle $\tilde{\beta}$ between neighboring pixels should be proportional to the difference of hue angle α between neighboring pixels. Namely, the Laplacian of $\tilde{\beta}$ should be the same as the Laplacian of α (Figure 4), scaled with a certain constant value.

$$\triangle\tilde{\beta}(j,i) = \triangle\alpha(j,i)\,. \tag{17}$$

This type of formula is called Poisson equation. From Eq. (13) and Eq. (16), Eq. (17) is represented as follows:

$$\begin{aligned}&\tilde{\beta}(j,i)-\frac{1}{4}\tilde{\beta}(j-1,i)-\frac{1}{4}\tilde{\beta}(j+1,i)\\&-\frac{1}{4}\tilde{\beta}(j,i-1)-\frac{1}{4}\tilde{\beta}(j,i+1)\\&=\frac{1}{4}\Delta\tilde{\alpha}(j-1,i)+\frac{1}{4}\Delta\tilde{\alpha}(j+1,i)\\&+\frac{1}{4}\Delta\tilde{\alpha}(j,i-1)+\frac{1}{4}\Delta\tilde{\alpha}(j,i+1)\,. \end{aligned} \tag{18}$$

The angle $\tilde{\beta}$ between neighboring pixels will become same as the angle α between neighboring pixels if we solve Eq. (18). Although $\tilde{\beta}$ becomes similar to α, the calculated $\tilde{\beta}$ becomes free from the cycle of $360°$. Unlike an identity equation $\tilde{\beta} = \alpha$ which copies the absolute angle, Eq. (17) preserves the relative angle among neighboring pixels.

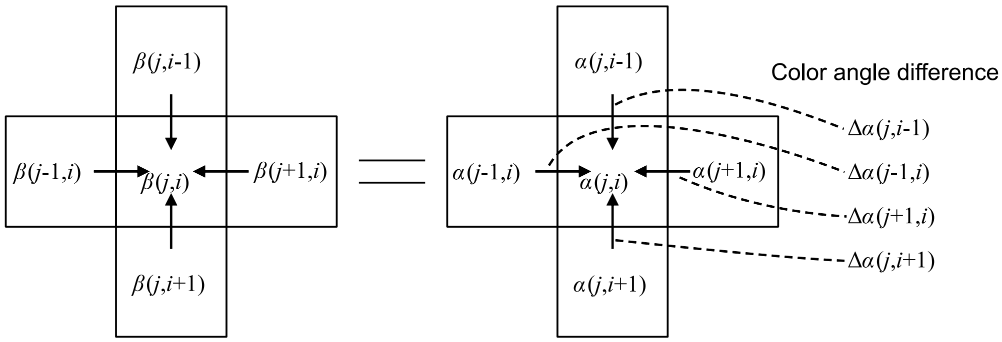


FIG. 4. Color difference between neighboring pixels.

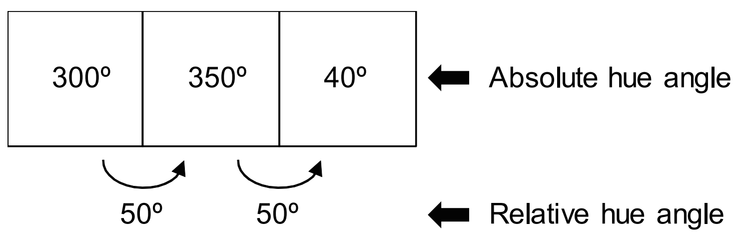


FIG. 5. Specific example of absolute and relative hue angles.

Suppose that the image consists of three pixels, and has hue angles α which are $300°$, $350°$, and $40°$ (Fig. 5). The color for trichromats will be blue, purple, and red. If we simply map these angles to the angle $\tilde{\beta}$, the color for dichromats becomes faint blue, deep blue, and yellow. However, if we solve the above mentioned Poisson equation, the calculated angle will be $300°$, $350°$, and $400°$. If we map these angles to the angle $\tilde{\beta}$, for example, to $150°$, $160°$, and $170°$, the color for dichromats becomes faint yellow, faint cyan, and faint blue. The color difference between neighboring pixels will be preserved if we solve the Poisson equation.

The closed-form solution to $\tilde{\beta}$ Eq. (18) can be obtained using the LU decomposition implemented in sparse matrix library.

The obtained $\tilde{\beta}$ is not in the required range, we normalize the standard deviation. We calculate the standard deviation σ of $\tilde{\beta}$ and the average μ. As for protanopia, we will adjust the average μ to be $170°$ and the standard deviation to be $15°$. As shown in Figure 6, we make the range $170° \pm 2\sigma$ to be equal to the range $140° \leq \beta \leq 200°$.

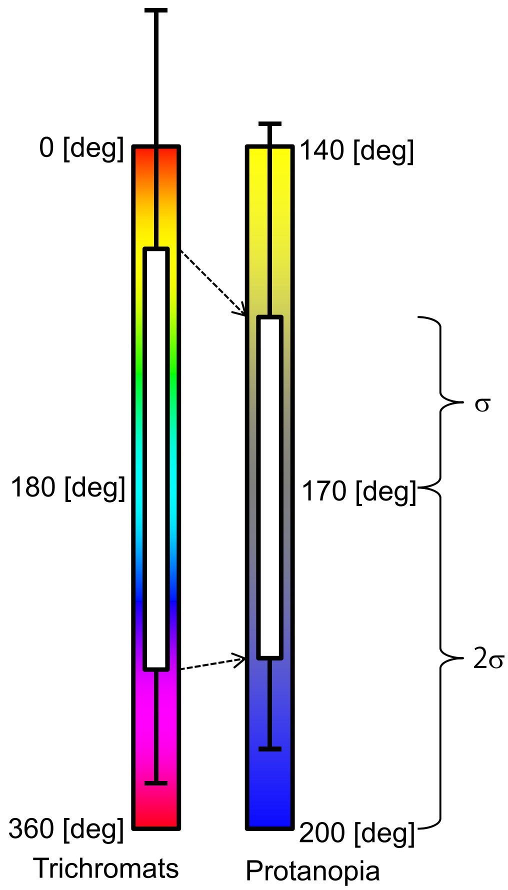


FIG. 6. Angle adjustment.

The hue angle $\tilde{\beta}$ for protanopia is modified as follows:

$$\beta = \frac{\pi}{180}\left(15\frac{\tilde{\beta}-\mu}{\sigma}+170\right) . \tag{19}$$

As for deuteranopia, we use $155°$ for μ and $7.5°$ for σ.

$$\beta = \frac{\pi}{180}\left(\frac{15}{2}\frac{\tilde{\beta}-\mu}{\sigma}+155\right) . \tag{20}$$

Now, we recover the XYZ value from the calculated hue angle β. For each pixel, we calculate the chromaticities x, y, and z from the hue angle β. Eq. (21) is used for protanopia and Eq. (22) is used for deutranopia.

$$\begin{aligned} x(j,i) &= \kappa_p \cos(\beta(j,i)) + 0.747\,, \\ y(j,i) &= \kappa_p \sin(\beta(j,i)) + 0.278\,, \\ z(j,i) &= 1.0 - x(j,i) - y(j,i)\,. \end{aligned} \tag{21}$$

$$x(j,i) = \kappa_b \cos(\beta(j,i)) + 1.000\,,$$
$$y(j,i) = \kappa_b \sin(\beta(j,i)) + 0.000\,,$$
$$z(j,i) = 1.0 - x(j,i) - y(j,i)\,. \tag{22}$$

Here, κ_p and κ_b are constant value set manually so that the calculated value will be inside the color gamut.

When we convert XYZ brightness of input image to xyz chromaticity, we divide by the sum, $\tilde{X} + \tilde{Y} + \tilde{Z}$. This value $\tilde{X} + \tilde{Y} + \tilde{Z}$ is again multiplied to xyz chromaticity of output image in order to convert it to represent the XYZ brightness. The output brightness $X(j,i)$, $Y(j,i)$, and $Z(j,i)$ of pixel (j,i) is calculated as Eq. (23) from the chromaticity $x(j,i)$, $y(j,i)$, and $z(j,i)$.

$$X(j,i) = x(j,i)\left(\tilde{X}(j,i) + \tilde{Y}(j,i) + \tilde{Z}(j,i)\right),$$
$$Y(j,i) = y(j,i)\left(\tilde{X}(j,i) + \tilde{Y}(j,i) + \tilde{Z}(j,i)\right),$$
$$Z(j,i) = z(j,i)\left(\tilde{X}(j,i) + \tilde{Y}(j,i) + \tilde{Z}(j,i)\right). \tag{23}$$

After that, we convert the XYZ value to the appearance of dichromats. Finally, we convert CIE-XYZ to sRGB.

IV. EXPERIMENT

A. Quantitative Experiment

In this section, we compare our method with existing methods[26,27]. The results of the color-based method[26], the gradient-based method[27], and the proposed method applied to the input image shown in Figure 7(a), are shown in Fig. 7(c), Fig. 7(d), and Fig. 7(e), respectively. Here, the results for deutranopia are shown. The dichromats' appearance is shown in Fig. 7(b), where all these three methods (Fig. 7(c), Fig. 7(d), and Fig. 7(e)) distinguished the color difference of red berry and green leaf.

Now, let us compare the performance of these three method numerically. The input image is shown in Figure 8(a), while its protanopia appearance is shown in Fig. 8(b). The color distribution of this image is evenly placed in color space, and thus existing methods which stretches the plotted pixels in color space do not work. Our result is shown in Fig. 8(e), which show color exaggeration. Our methods have higher performance than existing methods, which are based on such approaches.

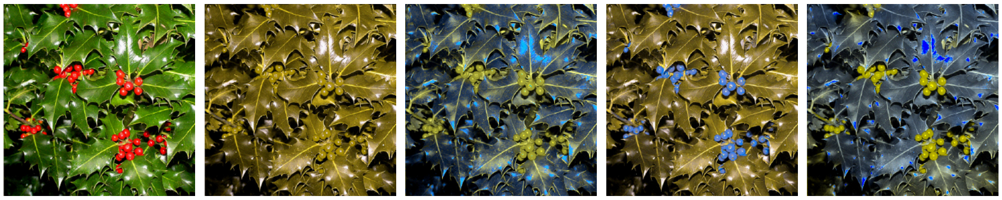


FIG. 7. Results of natural image: (a) Input image, (b) deutranopia image, (c) result of the color-based method, (d) result of the gradient-based method, and (e) our result.

One of the results[26], which is based on color space, is shown in Fig. 8(c). The result of other gradient-based method[27] is shown in Fig. 8(d). We calculated the color difference between point A and point B, as shown in Figure 9. Here, we use the chromaticity difference of $L*a*b*$. The $L*a*b*$ color space and its color difference $\sqrt{(a^*_A - a^*_B)^2 + (b^*_A - b^*_B)^2}$ are invalid for dichromats because it is designed for trichromats. However, we use it because there is no other choice. The difference is 15.97 for the existing color-based method[26], is 33.33 for the existing gradient-based method[27], and is 110.7 for our method, which shows the advantage of our method.

Another disadvantage of the previous color-based method[26] is the color gap indicated by the thick line in Fig. 9. This is because the method[26] use the absolute value of hue angle from $0°$ to $360°$, where the gap will cause between $360°$ and $0°$. On the other hand, we use the relative hue angle using the Poisson equation, and thus the color gap does not occur between $360°$ and $0°$.

The color around the center of the image varies from yellow to blue in Fig. 8(c) and Fig. 8(d). Fig. 8(c) and Fig. 8(d) have a single cycle of such color variation, while Fig. 8(e) haves two cycles of such color variation: the color varies from yellow to blue, blue to yellow, yellow to blue, and again blue to yellow in Fig. 8(e). This is the limitation of stretching the color space without nonlinear distortion (Fig. 8(c)). Also, the existing gradient-based method[27] only shifts the gradient of RGB (Fig. 8(d)), which does not consider the cyclic feature of chromaticity. Our method can be said that it has twice high performance than existing approaches.

B. Qualitative Experiment

Quantitative evaluation in Section IV A has shown that our method outperforms existing methods. However, one might be anxious about the gradation caused in Fig. 8 (e). In this section, we show some qualitative evaluation in order to show that this effect is not sensitive in natural images.

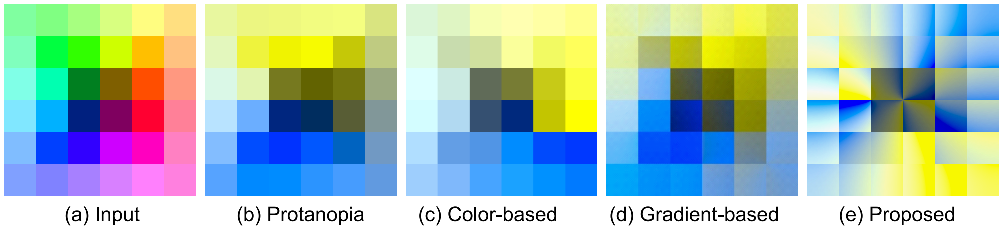


FIG. 8. Result of evenly distributed color: (a) input image, (b) protanopia appearance, (c) existing color-based method result, (d) existing gradient-based method result, and (e) proposed method result.

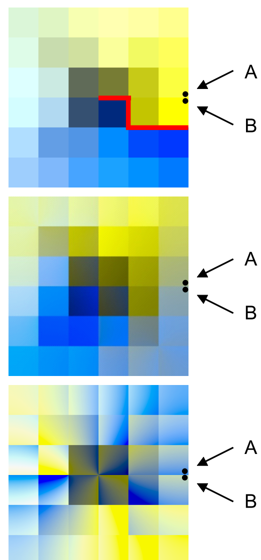


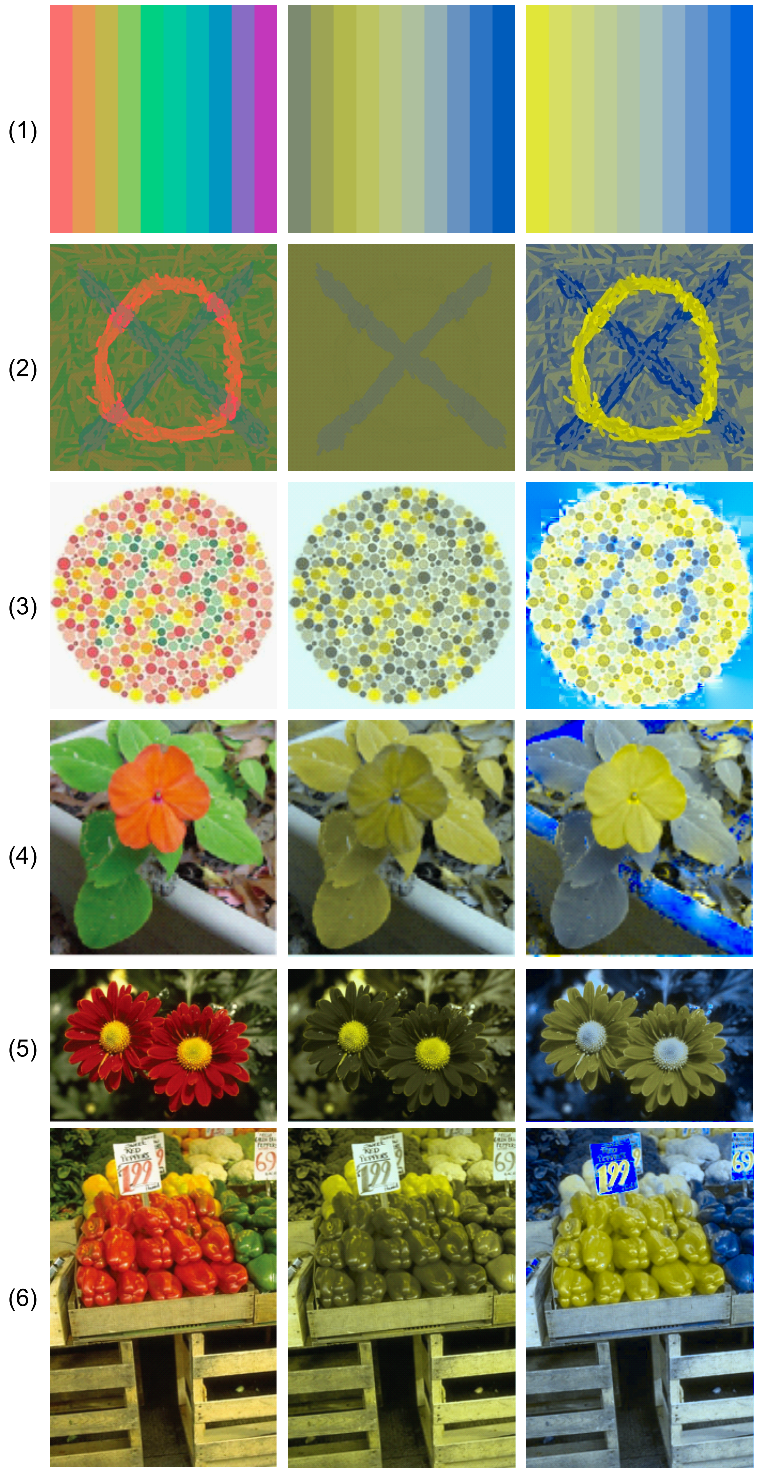
FIG. 9. Evaluation of color difference.

Figures 10 and 11 show some results. The color difference of our result resembles the trichromats' appearance, while the gradation artifacts are unnoticable.

V. DISCUSSION

Although the disadvantage of our method is the gradation effect as described in Sectio IV A, this disadvantage does not matter in natural image as described in Section IV B.

Another disadvantage of our method is shown in Figure 12. The color difference is not so exaggerated in this result. This is due to the characteristics of the proposed algorithm. Our method formulates an equation between neighboring pixels, and solves it. Therefore, if the colored region is separated by achromatic region, we cannot calculate the color difference between colored re-



(a) Input (b) Protanopia (c) Result

FIG. 10. Results of various images: (a) input image, (b) protanopia appearance, and (c) our result.

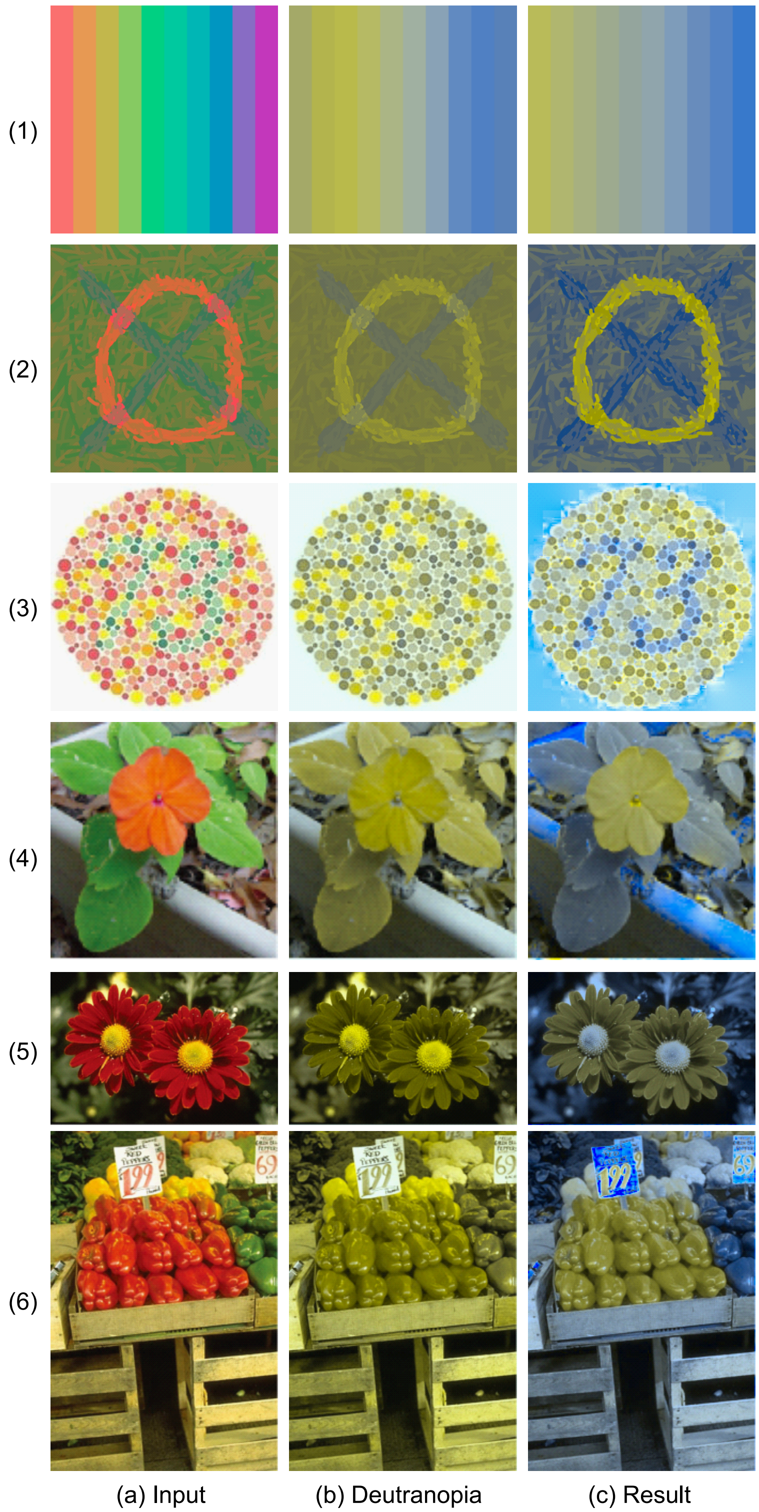


FIG. 11. Results of various images: (a) input image, (b) deutranopia appearance, and (c) our result.

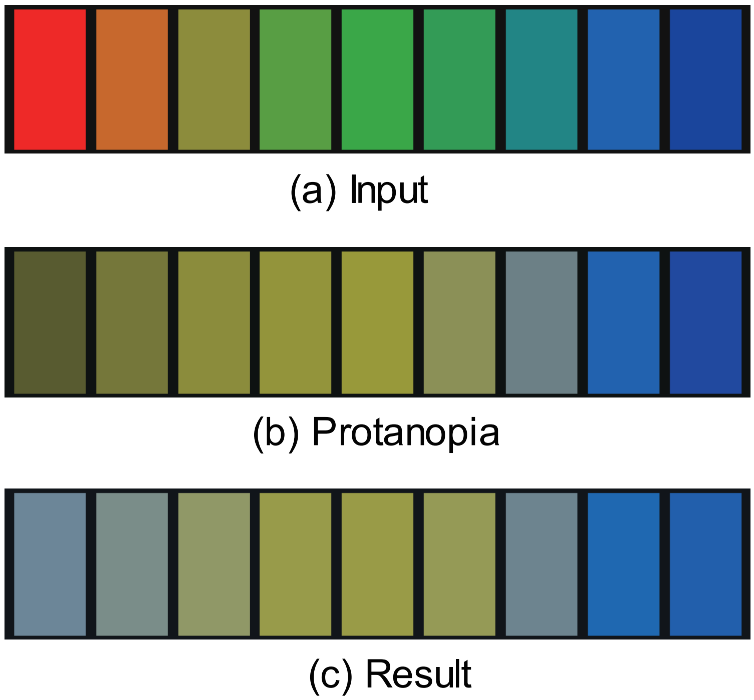


FIG. 12. Failure case: (a) input image, (b) protanopia appearance, and (c) our result.

gions. This problem is inevitable for our approach which analyzes local pixels. On the other hand, existing methods which analyzes global space of color are not sensitive to this problem. Mixing local approach and global approach will be our future plan for this work.

VI. CONCLUSION

In this article, we have proposed a method which enhance the visibility of dichromats. Our method converts the color of an image so that the image will be clear for dichromats. We have formulated the color difference of trichromat as a Poisson equation, and solved it to preserve the color difference, which can also be perceived by dichromats. The Poisson equation formulated in chromaticity space exaggerate the color difference of neighboring pixels, and at the same time, it preserves the chromaticity difference of trichromats. Experimental results show that our method is robust and beneficial. The disadvantage of our method is that it cannot be applied to achromatic images (black, gray, and white). Some kind of preprocessing or postprocessing may avoid such problems; however, such processings do not fundamentally solve the problem. We are planning to theoretically solve this problem in the future.

REFERENCES

[1]J. Bao, Y. Wang, Y. Ma, and X. Gu, "Re-coloring images for dichromats based on an improved adaptive mapping algorithm," *Int'l Conf. Audio, Language and Image Processing* (2008), pp. 152–156.

[2]S. Bao, G. Tanaka, H. Tamukoh, and N. Suetake, "Improvement of lightness modification method based on Craik-O'Brien effect for dichromats," *Int'l Symp. Intelligent Signal Processing and Communication Systems* (2015), pp. 75–78.

[3]W. Chen, W. Chen, and H. Bao, "An efficient direct volume rendering approach for dichromats," *IEEE Trans. Visual. Comput. Graph.* **17**, 2144–2152 (2011).

[4]M. Flinkman and S. Nakauchi, "Illuminations that improve color discrimination ability of people with red-green color vision deficiency," *J. Opt. Soc. Am. A* **34**, 1914–1923 (2017).

[5]J. Huang, Y. Tseng, S. Wu, and S. Wang, "Information preserving color transformation for protanopia and deuteranopia," *IEEE Signal Process. Lett.* **14**, 711–714 (2007).

[6]J.-B. Huang, S.-Y. Wu, and C.-S. Chen, "Enhancing color representation for the color vision impaired," *Workshop on Computer Vision Applications for the Visually Impaired* (2008).

[7]J. Huang, C. Chen, T. Jen, and S. Wang, "Image recolorization for the colorblind," *IEEE Int'l Conf. Acoustics, Speech and Signal Processing* (IEEE, Piscataway, NJ, 2009), pp. 1161–1164.

[8]C. Huang, K. Chiu, and C. Chen, "Temporal color consistency-based video reproduction for dichromats," *IEEE Trans. Multimedia* **13**, 950–960 (2011).

[9]M. Ichikawa, K. Tanaka, S. Kondo, K. Hiroshima, K. Ichikawa, S. Tanabe, and K. Fukami, "Preliminary study on color modification for still images to realize barrier-free color vision," *Int'l. Conf. Systems, Man and Cybernetics* (2004), pp. 36–41.

[10]J. Jeong, H. Kim, Y. Kim, T. Wang, and S. Ko, "Enhanced re-coloring method with an information preserving property for color-blind person," *Int'l. Conf. Consumer Electronics* (2012), pp. 600–601.

[11]Y. Ma, E. Wang, and Y. Wang, "An embedded image processing device for color vision deficiency," *Int'l. Conf. Biomedical Engineering and Informatics* (2011), pp. 1041–1045.

[12]M. Meng and G. Tanaka, "Proposal of minimization problem based lightness modification for protanopia and deuteranopia," *Int'l. Symp. Intelligent Signal Processing and Communication Systems* (2016), pp. 1–6.

[13]G. W. Meyer and D. P. Greenberg, "Color-defective vision and computer graphics displays," *IEEE Comput. Graph. Appl.* **8**, 28–40 (1988).

[14]M. M. Oliveira, "Towards more accessible visualizations for color-vision-deficient indificuals," *Comput. Sci. Engng.* **15**, 80–87 (2013).

[15]M. G. Ribeiro and A. J. P. Gomes, "A skillet-based recoloring algorithm for dichromats," *Int'l. Conf. e-Health Networking, Applications & Services* (2013), pp. 702–706.

[16]M. Ribeiro and A. J. P. Gomes, "Recoloring algorithms for colorblind people: a survey," *ACM Comput. Surv.* **52**, 37 (2019).

[17]J. Ruminski, J. Wtorek, J. Ruminska, M. Kaczmarek, A. Bujnowski, T. Kocejko, and A. Polinski, "Color transformation methods for dichromats," *3rd Conf. Human System Interactions* (2010), pp. 634–641.

[18]B. Sajadi, A. Majumder, M. M. Oliveira, R. G. Schneider, and R. Raskar, "Using patterns to encode color information for dichromats," *IEEE Trans. Vis. Comput. Graphics* **19**, 118–129 (2013).

[19]J. P. Srividhya, P. Sivakumar, and M. Rajaram, "The color blindness removal technique in image by using gradient map method," *Int'l. Conf. Signal Processing, Communication, Computing and Networking Technologies* (2011), pp. 24–29.

[20]G. Tennenholtz and I. Zachevsky, "Natural contrast enhancement for dichromats using similarity maps," *Int'l. Conf. Science of Electrical Engineering* (2016), pp. 1–5.

[21]F. Yan, G. Liu, and Y. Zheng, "An HTML5 based discoloration algorithm for people with low vision," *Int'l. Conf. Biomedical Engineering and Informatics* (2016), pp. 837–842.

[22]K. Rasche, R. Geist, and J. Westall, "Detail preserving reproduction of color images for monochromats and dichromats," *IEEE Comput. Graph. Appl.* **25**, 22–30 (2005).

[23]S. Nakauchi and T. Onouchi, "Detection and modification of confusing color combinations for red-green dichromats to achieve a color universal design," *Color Res. Appl.* **33**, 203–211 (2008).

[24]G. R. Kuhn, M. M. Oliveira, and L. A. F. Fernandes, "An efficient naturalness-preserving image-recoloring method for dichromats," *IEEE Trans. Vis. Comput. Graphics* **14**, 1747–1754 (2008).

[25]G. Tanaka, N. Suetake, and E. Uchino, "Lightness modification of color image for protanopia and deuteranopia," *Opt. Rev.* **17**, 14–23 (2010).

[26]D. Miyazaki, S. Taomoto, and S. Hiura, "Extending the visibility of dichromats using histogram equalization of hue value defined for dichromats," *Int. J. Image Graph.* **19**, 1950016, 13 (2019).

[27] J. T. Simon-Liedtke and I. Farup, “Multiscale daltonization in the gradient domain,” *J. Perceptual Imaging* **1**, 1–12 (2018).

[28] I. Farup, “Individualised halo-free gradient-domain color image daltonisation,” *J. Imaging* **6**, 10 (2020).

[29] D. B. Judd, “Standard response functions for protanopic and deuteranopic vision,” *J. Opt. Soc. Am.* **35**, 199–221 (1945).

[30] H. Brettel, F. Viénot, and J. D. Mollon, “Computerized simulation of color appearance for dichromats,” *J. Opt. Soc. Am. A* **14**, 2647–2655 (1997).